HIGHER

3.1 Vector Geometry: Proof and Ratio Problems: 30-minute Pack

Total Marks

EDEXCEL 4MA1 · Calculator allowed · about 30 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 OA = a and OB = b. The points P, Q and R have position vectors OP = 2a - 3b, OQ = 3a + 3b and OR = (9/2)a relative to O.
Show whether P, Q and R lie on a straight line. Give a reason.

(Total for Question 1 is 5 marks)

2 A parallelogram has vertices P, Q, R and S in that order. OP = (1, 0), OQ = (3, 2) and OR = (5, 5). Using vector methods, prove that PQRS is a parallelogram and find OS.

(Total for Question 2 is 6 marks)

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3 Points A, B and D have position vectors OA = (0, 0), OB = (6, 0) and OD = (2, 4). Point C lies on BD such that BC : CD = 1 : 2. Using vectors, find OC and then determine, giving a reason, whether ABCD is a parallelogram.

(Total for Question 3 is 6 marks)

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4 Points X, Y and Z have position vectors OX = (1, 1), OY = (4, 5) and OZ = (7, 9). A point T divides XZ in the ratio XT : TZ = 2 : 3. Using vectors, find OT and then show that YT is parallel to XZ and give the scalar factor.

(Total for Question 4 is 7 marks)

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Mark scheme · 3.1 Vector Geometry: Proof and Ratio Problems: 30-minute Pack

Question 1

PQ = OQ - OP	**M1**
PQ = a + 6b	**A1**
PR = OR - OP	**M1**
PR = (5/2)a + 3b	**A1**
PR is not a scalar multiple of PQ, so the points are not on a straight line	**B1**

Answer: Not collinear: PR is not a multiple of PQ

Question 2

Level 1 Identifies one vector calculation correctly, for example finds PQ or QR, but with incomplete justification. **1-2**

Level 2 Shows both relevant side vectors are equal or parallel, demonstrating the parallelogram property, and obtains OS form with minor algebraic errors allowed. **3-4**

Level 3 Completes a clear vector proof that PQ = SR (or QR = PS), deduces PQRS is a parallelogram, and finds OS correctly with clear reasoning. **5-6**

Indicative content:

- Compute PQ = OQ - OP = (3,2) - (1,0) = (2,2).
- Compute QR = OR - OQ = (5,5) - (3,2) = (2,3). PQ and QR are not parallel, so P, Q and R are not collinear and PQRS can be a genuine parallelogram.
- For PQRS to be a parallelogram PS = QR, so OS = OP + PS = OP + QR = (1,0) + (2,3) = (3,3).
- Check: SR = OR - OS = (5,5) - (3,3) = (2,2) = PQ, so PQ and SR are equal and parallel (and QR = PS = (2,3)), which proves PQRS is a parallelogram.
- Final answer: OS = (3, 3).

Question 3

Level 1 Finds an intermediate vector correctly, for example expresses OC in terms of OB and OD but with arithmetic errors. **1-2**

Level 2 Calculates OC correctly from the ratio and finds one pair of opposite side vectors, for example AB and DC, with minor omissions in justification. **3-4**

Level 3 Accurately finds OC using section formula, compares both pairs of opposite sides as vectors, and concludes that ABCD is not a parallelogram with clear reasoning. **5-6**

Indicative content:

- Parameterise C on BD: OC = OB + t(BD) where BD = OD - OB = (2,4) - (6,0) = (-4,4).
- Use BC : CD = 1 : 2 so C divides BD in ratio 1:2 starting from B, hence t = 1/3 (since BC is 1/3 of BD), so OC = OB + (1/3)(BD) = (6,0) + (1/3)(-4,4) = (6,0) + (-4/3,4/3) = (14/3, 4/3).
- Alternatively OC = (2 OB + 1 OD)/3 if using section formula along BD from B to D, giving OC = (2(6,0) + (2,4))/3 = ((12+2)/3, (0+4)/3) = (14/3, 4/3).
- Compute AB = OB - OA = (6,0) and DC = OC - OD = (14/3,4/3) - (2,4) = (14/3 - 6/3, 4/3 - 12/3) = (8/3, -8/3) so AB is not equal to DC.
- Also AD = OD - OA = (2,4) and BC = OC - OB = (14/3,4/3) - (6,0) = (-4/3,4/3), so AD is not equal to BC. Neither pair of opposite sides is equal as a vector (and B, C and D are collinear, so three of the four vertices lie on one line), hence ABCD is not a parallelogram.

Question 4

Level 1 Uses a section formula incorrectly or finds incomplete vector expressions for OT or YT. **1-2**

Level 2 Finds OT correctly and shows a relation between YT and XZ but may miss a clear scalar factor or provide weak justification for parallelism. **3-5**

Level 3 Calculates OT precisely using the ratio, demonstrates YT is a scalar multiple of XZ with correct scalar factor, and provides a clear vector argument for parallelism. **6-7**

Indicative content:

- Compute XZ = OZ - OX = (7,9) - (1,1) = (6,8).
- Since XT : TZ = 2 : 3, T divides XZ so OT = OX + (2/5) XZ = (1,1) + (2/5)(6,8) = (1,1) + (12/5,16/5) = (17/5,21/5).
- Compute YT = OT - OY = (17/5,21/5) - (4,5) = (17/5 - 20/5, 21/5 - 25/5) = (-3/5, -4/5).
- Compare YT with XZ: XZ = (6,8) and YT = (-1/10)(6,8). So YT = -1/10 XZ, showing YT is parallel to XZ with scalar factor -1/10, indicating opposite direction.
- Conclude YT is parallel to XZ, scalar factor -1/10, because one vector is a scalar multiple of the other.

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