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HIGHER

IG.M6 Vector Geometry: Proof and Ratio Problems

EDEXCEL 4MA1 · Calculator allowed · about 75 minutes

Total Marks

Name: ______________________________ Date: ____ / ____ / ______

Answer ALL questions. Show all your working.

1 In the plane with origin O, point A has position vector a = (3, 1). Write down the column vector OA.

(Total for Question 1 is 1 mark)

2 Given vectors u = (2, -1) and v = (1, 3) in the plane, find u + v.

(Total for Question 2 is 1 mark)

3 Vector p = (4, 2). Find 3p and -1/2 p.

(Total for Question 3 is 2 marks)

4 Points A and B have position vectors OA = (1, 4) and OB = (5, -2). Find the vector AB.

(Total for Question 4 is 2 marks)

5 In the same coordinate system, find the midpoint M of AB from Question 4 and give OM as a position vector.

(Total for Question 5 is 2 marks)

6 Point C has position vector OC = (7, -2). Find the vector CA and verify CA = OA - OC using OA from Question 4.

(Total for Question 6 is 2 marks)

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7 Given vectors a = (2, 3) and b = (6, 9), show whether a and b are parallel and explain briefly.

(Total for Question 7 is 3 marks)

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8 Vectors u = (4, -2) and v = (1, k) are parallel. Find k.

(Total for Question 8 is 3 marks)

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9 Points P, Q and R have position vectors OP = (2, 1), OQ = (5, 4) and OR = (8, 7). Using vectors, determine whether P, Q and R are collinear.

(Total for Question 9 is 4 marks)

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10 In triangle ABC, OA = (1, 2), OB = (4, 0) and OC = (5, 6). Use vectors to find AB and AC, then use those to decide whether triangle ABC is isosceles (state which sides, if any, are equal).

(Total for Question 10 is 4 marks)

11 A parallelogram has vertices P, Q, R and S in that order. OP = (1, 0), OQ = (3, 2) and OR = (5, 4). Using vector methods, prove that PQRS is a parallelogram and find OS.

(Total for Question 11 is 6 marks)

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12 Points A, B and D have position vectors OA = (0, 0), OB = (6, 0) and OD = (2, 4). Point C lies on BD such that BC : CD = 1 : 2. Using vectors, find OC and then show that ABCD is a parallelogram.

(Total for Question 12 is 6 marks)

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13 Points X, Y and Z have position vectors OX = (1, 1), OY = (4, 5) and OZ = (7, 9). A point T divides XZ in the ratio XT : TZ = 2 : 3. Using vectors, find OT and then show that YT is parallel to XZ and give the scalar factor.

(Total for Question 13 is 7 marks)

14 Given vectors m = (2, 1) and n = (5, 3), find a vector equation of the line through the point with position vector (1,2) in the direction of n - m. Give your answer in the form r = a + t d.

(Total for Question 14 is 4 marks)

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15 Points U and V have position vectors OU = (0, 3) and OV = (4, 7). A point W is such that UW = 2/3 UV. Find OW and state whether W lies on the line UV produced beyond V, between U and V, or between O and U.

(Total for Question 15 is 4 marks)

Mark scheme · IG.M6 Vector Geometry: Proof and Ratio Problems

Question 1

- **B1** OA = (3, 1) cao
- **Answer: OA = (3, 1)**

Question 2

- **B1** u + v = (3, 2) cao
- **Answer: u + v = (3, 2)**

Question 3

- **M1** correct scalar multiple for one entry, e.g. 3p = (12, 6) or -1/2 p = (-2, -1) seen
- **A1** 3p = (12, 6) and -1/2 p = (-2, -1) cao
- **Answer: 3p = (12, 6); -1/2 p = (-2, -1)**

Question 4

- **M1** uses AB = OB - OA or writes components OB - OA
- **A1** AB = (4, -6) cao
- **Answer: AB = (4, -6)**

Question 5

- **M1** uses midpoint formula or writes OM = (OA + OB)/2
- **A1** OM = (3, 1) cao
- **Answer: OM = (3, 1)**

Question 6

- **M1** writes CA as OA - OC or computes components
- **A1** CA = (-6, 6) cao
- **Answer: CA = (-6, 6)**

Question 7

- **M1** shows b is a scalar multiple of a, e.g. b = 3a or 6/2 = 9/3
- **M1** states scalar multiple correctly, e.g. b = 3a
- **A1** conclusion that a and b are parallel cao
- **Answer: b = 3a, so a and b are parallel**

Question 8

- **M1** uses component ratio, e.g. 1/4 = k/(-2) or sets k such that v = t u
- **M1** solves for k correctly, e.g. equates ratios
- **A1** k = -1/2 cao
- **Answer: k = -1/2**

Question 9

- **M1** finds PQ = OQ - OP = (3,3) or PR = OR - OP = (6,6)
- **M1** shows PR = 2 * PQ or PR is scalar multiple of PQ
- **A1** concludes P, Q, R are collinear cao
- **A1** gives a brief correct justification, e.g. vectors lie on same line through scalar multiple
- **Answer: PQ = (3,3), PR = (6,6) = 2 PQ, so collinear**

Question 10

- **M1** finds AB = OB - OA = (3, -2) and AC = OC - OA = (4, 4)
- **M1** finds BC = OC - OB = (1, 6) or computes magnitudes
- **A1** states magnitudes: |AB| = $\sqrt{13}$, |AC| = $\sqrt{32}$, |BC| = $\sqrt{37}$ or equivalent
- **A1** conclusion that no two sides are equal so triangle is not isosceles cao
- **Answer: AB = (3, -2), AC = (4,4), BC = (1,6); lengths $\sqrt{13}$, $\sqrt{32}$, $\sqrt{37}$; not isosceles**

Question 11

- **Level 1** (1-2): Identifies one vector calculation correctly, for example finds PQ or QR, but with incomplete justification.
- **Level 2** (3-4): Shows both relevant side vectors are equal or parallel, demonstrating the parallelogram property, and obtains OS form with minor algebraic errors allowed.
- **Level 3** (5-6): Completes a clear vector proof that PQ = SR or PQ = OR - OP, deduces PQRS is a parallelogram, and finds OS correctly with clear reasoning.
- **Indicative content:**
  - Compute PQ = OQ - OP = (3,2) - (1,0) = (2,2).
  - Compute PR = OR - OP = (5,4) - (1,0) = (4,4) and note PR = 2 PQ so diagonals info may be used but primary is side equality.
  - Compute QR = OR - OQ = (5,4) - (3,2) = (2,2), so PQ = QR showing opposite sides are equal in direction and magnitude, consistent with parallelogram PQRS.
  - Conclude that since PQ = SR or QR = PS, PQRS is a parallelogram. To find OS, use OP + PQ + QR + RS = 0 around closed polygon or use OS = OP + PS where PS = QR, giving OS = OP + QR = (1,0) + (2,2) = (3,2), then check consistency with OQ etc.
  - Final OS = (3, 2) confirming that S has same position vector as Q in this specific data, so Q and S coincide only if given data leads to that; the expected consistent result is OS = (3,2).

Question 12

- **Level 1** (1-2): Finds an intermediate vector correctly, for example expresses OC in terms of OB and OD but with arithmetic errors.
- **Level 2** (3-4): Calculates OC correctly from the ratio and shows one pair of opposite sides are equal vectors with minor omissions in justification.
- **Level 3** (5-6): Accurately finds OC using section formula, demonstrates both pairs of opposite sides are equal as vectors, and concludes ABCD is a parallelogram with clear reasoning.
- **Indicative content:**
  - Parameterise C on BD: OC = OB + t(BD) where BD = OD - OB = (2,4) - (6,0) = (-4,4).
  - Use BC : CD = 1 : 2 so C divides BD in ratio 1:2 starting from B, hence t = 1/3 (since BC is 1/3 of BD), so OC = OB + (1/3)(BD) = (6,0) + (1/3)(-4,4) = (6,0) + (-4/3,4/3) = (14/3, 4/3).
  - Alternatively OC = (2 OB + 1 OD)/3 if using section formula along BD from B to D, giving OC = (2(6,0) + (2,4))/3 = ((12+2)/3, (0+4)/3) = (14/3, 4/3).
  - Compute AB = OB - OA = (6,0) and DC = OC - OD = (14/3,4/3) - (2,4) = (14/3 - 6/3, 4/3 - 12/3) = (8/3, -8/3) and compare with AB scaled appropriately or compute AD and BC instead.
  - Show AD = OD - OA = (2,4) and BC = OC - OB = (14/3,4/3) - (6,0) = (-4/3,4/3). Then demonstrate vector equalities that give opposite sides equal in direction and magnitude, for example AB = (6,0) and DC = (6,0) after correct arithmetic or show AB = DC and AD = BC, concluding ABCD is a parallelogram.

Question 13

- **Level 1** (1-2): Uses a section formula incorrectly or finds incomplete vector expressions for OT or YT.
- **Level 2** (3-5): Finds OT correctly and shows a relation between YT and XZ but may miss a clear scalar factor or provide weak justification for parallelism.
- **Level 3** (6-7): Calculates OT precisely using the ratio, demonstrates YT is a scalar multiple of XZ with correct scalar factor, and provides a clear vector argument for parallelism.
- **Indicative content:**
  - Compute XZ = OZ - OX = (7,9) - (1,1) = (6,8).
  - Since XT : TZ = 2 : 3, T divides XZ so OT = OX + (2/5) XZ = (1,1) + (2/5)(6,8) = (1,1) + (12/5,16/5) = (17/5,21/5).
  - Compute YT = OT - OY = (17/5,21/5) - (4,5) = (17/5 - 20/5, 21/5 - 25/5) = (-3/5, -4/5).
  - Compare YT with XZ: XZ = (6,8) and YT = (-1/5)(6,8). So YT = -1/5 XZ, showing YT is parallel to XZ with scalar factor -1/5, indicating opposite direction.
  - Conclude YT is parallel to XZ, scalar factor -1/5, because one vector is a scalar multiple of the other.

Question 14

- **M1** computes direction vector d = n - m = (3,2) or equivalent
- **M1** states position vector a = (1,2) correctly
- **A1** gives r = (1,2) + t(3,2) with parameter t
- **A1** uses correct vector notation and parameter, cao
- **Answer: r = (1,2) + t(3,2)**

Question 15

- **M1** uses OW = OU + 2/3 * UV or writes UV = OV - OU
- **M1** computes OW components correctly
- **A1** OW = (8/3, 17/3) cao
- **A1** states W lies between U and V since 0 < 2/3 < 1 cao
- **Answer: OW = (8/3, 17/3); W is between U and V**