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HIGHER

IG.M7 The Sine Rule, Cosine Rule and Area of a Triangle

EDEXCEL 4MA1 · Calculator allowed · about 75 minutes

Total Marks

Name: ______________________________ Date: ____ / ____ / ______

Show full working for multi-step questions. Write answers in clear sentences for questions where a reason is asked. Suggested time 75 minutes.

1 In triangle ABC (not right-angled) angle A = 50 degrees, side a = BC = 12 cm and side b = AC = 15 cm. Use the sine rule to find the possible value(s) of angle B in degrees, giving answers to the nearest degree. State if two values are possible.

(Total for Question 1 is 2 marks)

2 Triangle PQR has sides p = QR = 9 cm, q = RP = 14 cm and angle P = 40 degrees. Calculate the length r = PQ, correct to 2 decimal places, using the cosine rule.

(Total for Question 2 is 2 marks)

3 In triangle ABC, a = BC = 10 cm and angle A = 30 degrees. No other sides or angles are given. State briefly whether the sine rule can be used to find side b = AC uniquely, and explain if an ambiguous case may arise.

(Total for Question 3 is 1 mark)

4 Triangle DEF has sides DE = 8 cm, EF = 7 cm and DF = 10 cm. Use the cosine rule to find angle E (the angle at vertex E between DE and EF), giving the answer to the nearest degree.

(Total for Question 4 is 2 marks)

5 A triangle has two sides of lengths 13 cm and 14 cm with included angle 58 degrees between them. Work out the area of this triangle in square centimetres, correct to 1 decimal place, using the area formula Area = 1/2 ab sin C.

(Total for Question 5 is 3 marks)

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6 Triangle GHI has GH = 11 cm, HI = 9 cm and angle G = 46 degrees (angle at G between GH and GI). Calculate the third side GI to 2 decimal places using the cosine rule.

(Total for Question 6 is 3 marks)

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7 A boat sails from point A on a bearing of 075 degrees for 8.5 km to point B. From B it sails on a bearing of 200 degrees for 6 km to point C. Work out the straight-line distance AC, correct to 2 decimal places, by extracting the triangle ABC and using the cosine rule.

(Total for Question 7 is 2 marks)

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8 In triangle XYZ, side x = YZ = 7 cm, side y = ZX = 12 cm and angle Z = 38 degrees. Use the sine rule to find angle X in degrees, to the nearest degree. Show any ambiguous case consideration and give the correct final angle(s).

(Total for Question 8 is 3 marks)

9 A triangular prism has an equilateral triangular cross-section with side 6 cm. A triangle is formed by one edge of length 6 cm on a triangular face and a line of length 8 cm from one end of that edge to a point elsewhere on the prism, with an angle of 60 degrees between the 6 cm edge and the 8 cm line. Work out the area of that triangle using 1/2 ab sin C and give your answer correct to 1 decimal place.

(Total for Question 9 is 3 marks)

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10 Triangle ABC has sides AB = 10 cm, AC = 14 cm and angle A = 47 degrees. First use the cosine rule to find BC to 2 decimal places, then find the area of triangle ABC to 1 decimal place using 1/2 ab sin C (you will need an appropriate angle).

(Total for Question 10 is 3 marks)

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11 Show that for triangle MNO with sides MN = 7 cm, NO = 13 cm and MO = 15 cm, the angle at N is obtuse. Use the cosine rule and give the angle to the nearest degree.

(Total for Question 11 is 2 marks)

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12 A triangular sail has sides 5.2 m and 7.4 m with included angle 112 degrees. A designer needs the area to the nearest 0.1 m^2, and also the length of the third side to 2 decimal places. First work out the area using 1/2 ab sin C, then find the third side using the cosine rule.

(Total for Question 12 is 4 marks)

13 Triangle RST has RS = 9 cm, ST = 16 cm and angle R = 41 degrees. Use the sine rule to find angle T to the nearest degree, then find the area of triangle RST to 1 decimal place.

(Total for Question 13 is 4 marks)

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14 A surveyor measures a triangle formed by three stakes A, B and C. He finds AB = 21 m, AC = 13 m and angle A = 102 degrees. (a) Calculate BC to 2 decimal places using the cosine rule. (b) Calculate the area of triangle ABC to 1 decimal place. Show all working.

(Total for Question 14 is 6 marks)

Mark scheme · IG.M7 The Sine Rule, Cosine Rule and Area of a Triangle

Question 1

- **M1** substitutes into sine rule: sin B / 15 = sin 50 / 12 and evaluates sin B = 15*sin50/12
- **A1** gives both possible angles B = 73 degrees and B = 107 degrees, correct to nearest degree, with statement that two values are possible
- **Answer: B = 73 degrees or 107 degrees (to nearest degree), two possible values**

Question 2

- **M1** uses $r^2 = p^2 + q^2 - 2*p*q*\cos P$ with correct substitution: $r^2 = 9^2 + 14^2 - 2*9*14*\cos 40$
- **A1** r = 9.16 cm to 2 decimal places, cao
- **Answer: r = 9.16 cm**

Question 3

- **B1** correct statement that b cannot be determined uniquely from only a and A and that an ambiguous case may arise
- **Answer: Not unique: the sine rule does not give b from only a and A; an ambiguous case can arise and additional information is needed**

Question 4

- **M1** uses $\cos E = (DE^2 + EF^2 - DF^2) / (2*DE*EF)$ with correct substitution
- **A1** E = 83 degrees (nearest degree), cao
- **Answer: E = 83 degrees**

Question 5

- **M1** substitutes into area formula: Area = 1/2 * 13 * 14 * sin 58
- **M1** evaluates sine and multiplies correctly to a suitable intermediate accuracy
- **A1** gives final area = 77.2 cm^2 correct to 1 decimal place, cao
- **Answer: 77.2 cm^2**

Question 6

- **M1** applies cosine rule to form equation: $HI^2 = GH^2 + GI^2 - 2*GH*GI*\cos G$ and rearranges to a quadratic in GI
- **M1** solves the quadratic correctly for GI (algebraic or using quadratic formula)
- **A1** gives GI = 11.93 cm and GI = 3.35 cm to 2 decimal places, or notes the appropriate physically valid value(s) with correct rounding
- **Answer: GI = 11.93 cm or GI = 3.35 cm (to 2 dp)**

Question 7

- **M1** correctly finds the included angle at B between BA and BC: the back-bearing from B to A is 075 + 180 = 255 degrees, and the bearing from B to C is 200 degrees, so angle ABC = 255 - 200 = 55 degrees
- **A1** uses cosine rule $AC^2 = AB^2 + BC^2 - 2*AB*BC*\cos 55$ and gives AC = 7.05 km to 2 dp, cao
- **Answer: AC = 7.05 km**

Question 8

- **M1** recognises this is an SAS configuration (angle Z is included between sides x = YZ = 7 and y = ZX = 12), so the cosine rule must be used first to find the third side z = XY: $z^2 = x^2 + y^2 - 2xy \cos Z = 7^2 + 12^2 - 2*7*12*\cos 38$
- **M1** evaluates z = XY = 7.79 cm (to 2 dp), then applies the sine rule sin X / x = sin Z / z to get sin X = 7*sin38/7.79
- **A1** gives X = 34 degrees (to nearest degree), cao
- **Answer: X = 34 degrees (to nearest degree)**

Question 9

- **M1** identifies the triangle sides to use: face edge = 6 cm, line to the third point = 8 cm, and the given angle of 60 degrees between them
- **M1** substitutes into area formula Area = 1/2 * 6 * 8 * sin60 and evaluates correctly
- **A1** gives area = 20.8 cm^2 correct to 1 decimal place, cao
- **Answer: 20.8 cm^2**

Question 10

- **M1** uses cosine rule to find $BC^2 = AB^2 + AC^2$ - 2*AB*AC*cos A with correct substitution
- **M1** evaluates BC to 2 dp correctly
- **A1** uses area formula with two sides and included angle (e.g. 1/2 * AB * AC * sin A) and gives area to 1 dp correctly
- **Answer: BC = 10.25 cm (to 2 dp), Area = 51.2 cm^2 (to 1 dp)**

Question 11

- **M1** uses cosine rule: cos N = ($MN^2 + NO^2 - MO^2$) / (2*MN*NO) with correct substitution
- **A1** evaluates cos N and finds N = 92 degrees (nearest degree), concluding angle N is obtuse
- **Answer: Angle N = 92 degrees (to nearest degree), so it is obtuse**

Question 12

- **M1** uses area formula: 1/2 * 5.2 * 7.4 * sin112 with correct substitution
- **M1** evaluates area to required accuracy
- **M1** uses cosine rule to form third $side^2 = 5.2^2 + 7.4^2$ - 2*5.2*7.4*cos112
- **A1** gives third side = 10.52 m (to 2 dp) and area = 17.8 m^2 (to 1 dp), cao
- **Answer: Area = 17.8 m^2, third side = 10.52 m**

Question 13

- **M1** uses sine rule sin T / RS = sin R / ST or equivalent correctly and forms sin T = RS*sin R / ST = 9*sin41/16
- **M1** evaluates sin T = 0.369, giving angle T = 22 degrees (nearest degree); notes this is the only valid solution since RS < ST means angle T < angle R, so the supplementary angle would make the angle sum exceed 180 degrees
- **M1** finds angle S = 180 - 41 - 22 = 117 degrees, then uses area formula 1/2*RS*ST*sin S with correct substitution
- **A1** gives angle T = 22 degrees and area = 64.0 cm^2 (answers to required accuracy), cao
- **Answer: T = 22 degrees, Area = 64.0 cm^2**

Question 14

- **M1** uses cosine rule $BC^2 = AB^2 + AC^2$ - 2*AB*AC*cos A with correct substitution
- **M1** evaluates BC to 2 dp correctly
- **M1** uses area formula Area = 1/2 * AB * AC * sin A with correct substitution
- **M1** evaluates sine and multiplies to a suitable intermediate accuracy
- **A1** gives BC = 26.90 m to 2 dp, cao
- **A1** gives area = 133.5 m^2 to 1 dp, cao
- **Answer: BC = 26.90 m, Area = 133.5 m^2**