



IG.M9 Arithmetic and Geometric Sequences and Series

EDEXCEL 4MA1 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / ____

Answer ALL questions. Show all your working.

- 1** In a savings plan the monthly deposit follows an arithmetic sequence with first term $a = 7$ and common difference $d = 5$. Find the n th term of this arithmetic sequence, written in terms of n .

(Total for Question 1 is 1 mark)

- 2** An arithmetic sequence used to model temperatures has n th term $a_n = 12 - 4n$. Find the first three terms of this sequence, giving each term explicitly.

(Total for Question 2 is 1 mark)

- 3** A shop reduces the price of an item by 6 pounds each week. The price is 90 pounds at week 1. Treating the weekly prices as an arithmetic sequence, find the price at week 10 and state the units.

(Total for Question 3 is 2 marks)

- 4** Calculate the sum of the first 8 terms of the arithmetic sequence 3, 7, 11, ... using the arithmetic series formula. State your answer as an exact integer.

(Total for Question 4 is 2 marks)

- 5 Find the least value of n such that the partial sum of the arithmetic series with first term $a = 5$ and common difference $d = 3$ exceeds 500. Use $S_n = n/2(2a + (n - 1)d)$ and show your working.

(Total for Question 5 is 3 marks)

- 6 A geometric sequence used to model population has first term $a = 6$ and common ratio $r = 1/3$. Find the general n th term $a_n = ar^{n-1}$ and evaluate a_4 , giving an exact fractional answer.

(Total for Question 6 is 2 marks)

- 7 Calculate the sum of the first 6 terms of the geometric sequence with first term $a = 8$ and common ratio $r = 1/2$, using the finite geometric series formula $S_n = a(1 - r^n)/(1 - r)$. Give your answer as a fraction and a decimal.

(Total for Question 7 is 3 marks)

- 8 A convergent geometric series models a payment stream with first term $a = 5$ and common ratio $r = 0.6$. Find the sum to infinity of this series, using $S_{\infty} = a/(1 - r)$. Give your answer with one decimal place.

(Total for Question 8 is 2 marks)

- 9 A population is modelled by a geometric decay where the initial population is 2500 and it decreases by 12% each year. Find the population after 5 years, giving your answer to the nearest whole number and using ar^{n-1} with a as the first-year population.

(Total for Question 9 is 2 marks)

- 10** A convergent geometric series has sum to infinity 75 and first term 5. Find the common ratio r of this series, giving r as an exact fraction and as a decimal to 4 decimal places.

(Total for Question 10 is 3 marks)

- 11** For the geometric sequence with first term $a = 3$ and common ratio $r = \frac{4}{5}$, calculate the sum of the first 6 terms. Give your answer exactly as a fraction and as a decimal to 3 decimal places.

(Total for Question 11 is 3 marks)

- 12** An arithmetic sequence models weekly savings: first term 20 pounds, common difference -2 pounds (weekly withdrawal). Calculate the sum of the first 10 terms and give the units.

(Total for Question 12 is 2 marks)

- 13** A geometric series has first term $a = 6$ and common ratio $r = \frac{1}{2}$. Find the integer value of n for which the sum of the first n terms equals 11.25. Show your algebraic steps.

(Total for Question 13 is 3 marks)

- 14** Show that the geometric series with first term $a = 2$ and common ratio $r = -\frac{1}{3}$ converges, and find its sum to infinity. Give the exact value and a decimal to two decimal places.

(Total for Question 14 is 4 marks)

Mark scheme · IG.M9 Arithmetic and Geometric Sequences and Series

Question 1

- **B1** nth term = $7 + (n - 1)5 = 5n + 2$, cao
- **Answer:** $5n + 2$

Question 2

- **B1** terms: 8, 4, 0 cao
- **Answer:** 8, 4, 0

Question 3

- **M1** uses $a_n = a + (n - 1)d$ with $a = 90$, $d = -6$ and $n = 10$
- **A1** $a_{10} = 90 + 9(-6) = 36$ pounds cao
- **Answer:** 36 pounds

Question 4

- **M1** uses $S_n = n/2(2a + (n - 1)d)$ with $n = 8$, $a = 3$, $d = 4$
- **A1** $S_8 = 8/2(2*3 + 7*4) = 4(6 + 28) = 136$ cao
- **Answer:** 136

Question 5

- **M1** writes inequality $n/2(2*5 + (n - 1)3) > 500$ and expands to a quadratic, e.g. $(3/2)n^2 + (7/2)n - 500 > 0$, oe
- **M1** shows testing or solving leads to $n = 17$ giving $S_{17} = 493 \leq 500$ and $n = 18$ giving $S_{18} = 549 > 500$
- **A1** least integer $n = 18$ cao
- **Answer:** $n = 18$

Question 6

- **M1** writes $a_n = 6*(1/3)^{n-1}$ and substitutes $n = 4$
- **A1** $a_4 = 6*(1/3)^3 = 6*(1/27) = 2/9$ cao
- **Answer:** $a_n = 6*(1/3)^{n-1}$, $a_4 = 2/9$

Question 7

- **M1** uses $S_6 = 8(1 - (1/2)^6)/(1 - 1/2)$
- **M1** evaluates numerator $1 - 1/64 = 63/64$ and divides by $1/2$ giving $8*(63/64)*2$
- **A1** $S_6 = 63/4 = 15.75$ cao
- **Answer:** $63/4$ or 15.75

Question 8

- **M1** uses $S_{\infty} = 5/(1 - 0.6)$
- **A1** $S_{\infty} = 5/0.4 = 12.5$ cao
- **Answer:** 12.5

Question 9

- **M1** uses $a_5 = 2500*(0.88)^5$ with $r = 0.88$ and $n = 5$
- **A1** population approximately 1319 (to nearest whole number) cao
- **Answer:** Approximately 1319 people

Question 10

- **M1** uses formula $75 = 5/(1 - r)$ and rearranges to $1 - r = 5/75 = 1/15$
- **M1** finds $r = 1 - 1/15 = 14/15$
- **A1** $r = 14/15$, approximately 0.9333 cao
- **Answer:** $r = 14/15$, approximately 0.9333

Question 11

- **M1** uses $S_6 = 3(1 - (4/5)^6)/(1 - 4/5)$
- **M1** evaluates $(4/5)^6 = 4096/15625$ and forms $S_6 = 3*(11529/15625)/(1/5) = 3*(11529/15625)*5$
- **A1** $S_6 = 34587/3125$, approximately 11.068 cao
- **Answer: 34587/3125, approximately 11.068**

Question 12

- **M1** uses $S_{10} = 10/2(2*20 + 9*(-2))$
- **A1** $S_{10} = 110$ pounds cao
- **Answer: 110 pounds**

Question 13

- **M1** uses $S_n = 6(1 - (1/2)^n)/(1 - 1/2) = 12(1 - (1/2)^n)$ and sets equal to 11.25
- **M1** solves $12(1 - (1/2)^n) = 11.25$ to get $1 - (1/2)^n = 0.9375$ and hence $(1/2)^n = 0.0625$
- **A1** $n = 4$ cao
- **Answer: n = 4**

Question 14

- **M1** states condition for convergence $|r| < 1$ and shows $|-1/3| = 1/3 < 1$
- **M1** uses $S_{\infty} = a/(1 - r)$ with $a = 2$ and $r = -1/3$
- **A1** exact sum $S_{\infty} = 2/(1 + 1/3) = 2/(4/3) = 3/2$ cao
- **A1** decimal 1.50 cao
- **Answer: $S_{\infty} = 3/2 = 1.50$**